Warm-Up

Rational Functions

1. What is the solution of $\left(\sqrt{x+2}\right)^2 = (x)^2$?

(#23)

SQ BOTH SIDES

SET = 0

$x+2 = x^2$

$-x-2 \quad -x-2$

$0 = x^2 - x - 2$

FACTOR

C, +B

$0 = (x-2)(x+1)$

$x = 2 \quad x = -1$

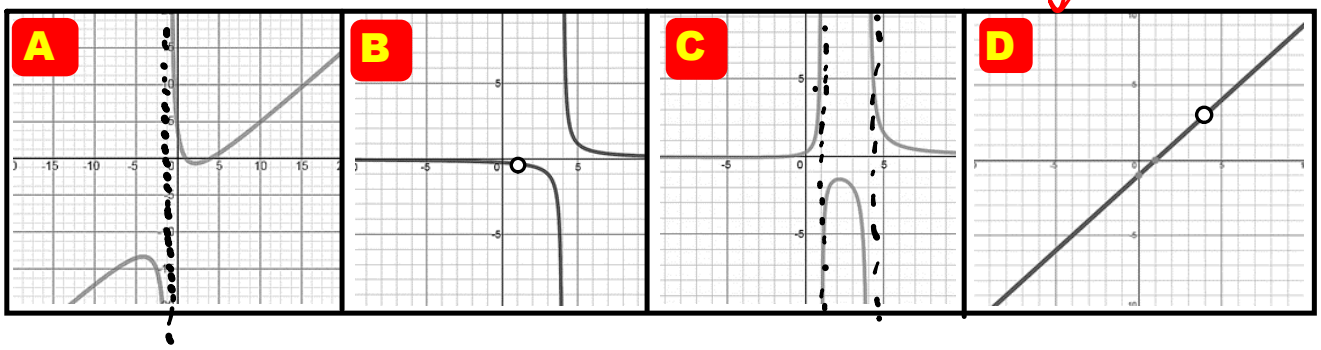
check takes the principal root +

* form of simplifying

ZPP

CHECK FOR EXTRANEOUS SOL.

2. Match each graph to the equation that best matches it.



F $y = \dfrac{x^2 - 5x + 4}{x - 4}$ $\Rightarrow \dfrac{0^2 - 5(0) + 4}{0 - 4} = -1$ $(0, -1)$

$y = \dfrac{(x-4)(x-1)}{x-4}$ D: $x \neq 4$ ↑ hole

weird graph

G $y = \dfrac{x+4}{x^2 - 5x + 4}$ $\Rightarrow x = 0,\ y = 1$ $(0,1)$

$y = \dfrac{x+4}{(x-4)(x-1)}$ D: $x \neq 4$, $x \neq 1$ ↑ VA

H $y = \dfrac{x^2 - 5x + 4}{x + 1}$ $x = 0,\ y = 4$ $(0,4)$

$y = \dfrac{(x-4)(x-1)}{x+1}$ D: $x \neq -1$ ↑ VA

I $y = \dfrac{x - 1}{x^2 - 5x + 4}$ $x = 0,\ y = -\frac{1}{4}$ $(0, -\frac{1}{4})$

$y = \dfrac{x-1}{(x-1)(x-4)}$ D: $x \neq 1$ ← hole, $x \neq 4$ ← VA

To add & subtract rational expressions, you can use much of what you know about adding & subtracting fractions.

The least common multiple, LCM, of the denominators of two rational expressions is also the Least Common Denominator, LCD, of the expressions. You can use the LCD to add and subtract rational expressions.

~~&~~ to +/− must have the same denominator

Key Concept

To find the LCM of several expressions, factor the expressions (numbers or polynomials) completely. The LCM is the product of the prime factors, each raised to the greatest power that occurs in any of the expressions.

Ex LCM: mental math

15 10

LCM: 30

18 30

LCM: 90

KC

15 → 3 · 5

18 → 2 · 5

LCM: $3 \cdot 2 \cdot 5 = 30$

18 → 3 · 6 → 3 · 3 · 2

30 → 3 · 10 → 3 · 2 · 5

LCM: $3^2 \cdot 2 \cdot 5 = 90$

Mechanics

To find the sum or difference of rational expressions with unlike denominators:

1. completely factor each denominator
2. identify the least common denominator, or LCD or LCM
3. multiply each expression by the factors needed to produce the LCD
4. add or subtract numerators
5. simplify

Problem 1 Finding the Least Common Multiple

KC #1

What is the LCM of the expressions?

a. $2x + 4$ and $x^2 - x - 6$

Step 1 Find the prime factors of each expression.

$2x+4$

find GCF

$2(x+2)$

x^2-x-6

$\cdot c \quad +b$

$(x-3)(x+2)$

Step 2 Write the product of the prime factors, each raised to the greatest power that occurs in either expression.

LCM:

$2(x+2)(x-3)$

unnecessary to write in standard form

How do you determine the exponent of each factor for the LCM?
Use the exponent from the expression that has that factor to the greatest power.

You Try

What is the LCM of the expressions?

b. $x^2 + 3x - 4$, $x^2 + 2x - 8$, and $x^2 - 4x + 4$

S1: factor $(x+4)(x-1)$ $(x+4)(x-2)$ $(x-2)(x-2)$

S2: LCM KC #1 $(x+4)(x-1)(x-2)^2$

Problem 2 Adding Rational Expressions

What is the sum of the two rational expressions in simplest form? State any restrictions on the variable. $\frac{x}{x-1} + \frac{2x-1}{x^2-3x+2}$

Find LCD

2 +3

$\frac{x}{x-1} + \frac{2x-1}{(x-2)(x-1)}$

Factor the denominators.

Rewrite each expression with the LCD.

LCM: $(x-1)(x-2)$

Add the numerators. Combine like terms.

$\frac{(x-2)}{(x-2)} \cdot \frac{x}{x-1} + \frac{2x-1}{(x-2)(x-1)}$ $\quad 2 \cdot \frac{1}{2} + \frac{3}{4}$ $\quad$ LCD: 4

Factor the numerator and divide out the common factors.

cancel add num.

$\frac{[x^2-2x}{(x-1)(x-2)} + \frac{2x-1]}{(x-2)(x-1)}$ $\quad \frac{2}{4} + \frac{3}{4}$

$\frac{x^2-1}{(x-1)(x-2)}$ → simplify 9-2

$\frac{(x-1)(x+1)}{(x-1)(x-2)} = \boxed{\frac{x+1}{x-2}}$

* factor
* eliminate

restrictions look all den

$x \neq 2$

$x \neq 1$

Problem 3 Subtracting Rational Expressions

What is the difference of the two rational expressions in simplest form? State any restrictions on the variable. $\frac{x+2}{x^2-2x} - \frac{x+2}{2x-4}$

GCF: x GCF: 2

$\frac{x+2}{x(x-2)} - \frac{x+2}{2(x-2)}$

① Factor den.

LCM: $2x(x-2)$

② Find LCD

$\frac{2 \cdot (x+2)}{2 \cdot x(x-2)} - \frac{(x+2) \cdot x}{2(x-2) \cdot x}$

③ Rewrite w. LCD

$\frac{2x+4}{2x(x-2)} - \frac{x^2+2x}{2x(x-2)}$

④ Simplify numerator

$2x+4-1(x^2+2x)$

$2x+4-x^2-2x$

$\frac{-x^2+4}{2x(x-2)}$

⑤ simplify

* factor
* eliminate

$\frac{-(x^2-4)}{2x(x-2)} = \frac{-(x+2)(x-2)}{2x(x-2)}$

$\frac{-(x+2)}{2x}$

restrictions $x \neq 0$ $x \neq 2$