Unit 7:
Radical Functions and Rational Exponents

7-1A Properties of Exponents
Pop Quiz
7-1 Roots and Radical Expressions
7-2 Multiplying and Dividing Radical Expressions
7-3 Binomial Radical Expressions
Mid-Chapter Review
Mid-Chapter Quiz
7-4 Rational Exponents
7-5 Solving Square Root and Other Radical Equations
Quiz
7-6 Function Operations
7-7 Inverse Relations and Functions
Chapter Review
Chapter Test

Function Operations

Today's Objectives

Algebra Objective: To add, subtract, multiply, and divide functions. To find the composite of two functions.

Language Objective: To identify in writing which parts of a function determine the domain. To explain in writing how composition functions are used to make purchasing decisions.

A2.L7-6.NOTES

BIG IDEA

take note

Key Concepts **Function Operations**

Addition	$(f + g)(x) = f(x) + g(x)$
Subtraction	$(f - g)(x) = f(x) - g(x)$
Multiplication	$(f \cdot g)(x) = f(x) \cdot g(x)$
Division	$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$

The domains of the sum, difference, product, and quotient functions consist of the x-values that are in the domains of *both* f and g. Also, the domain of the quotient function does not contain any x-value for which $g(x) = 0$.

★ Reminder

$y = 3x + 2$ — Output, Input

$f(x) = 3x + 2$ — Read as "f of x" or "function f of x."

$f(1) = 3(1) + 2$ — "f of 1" is the output when 1 is the input.

For $f(x) = -4x + 1$, what is the output for the given input?

a. -2 $f(-2) = -4(-2) + 1 = 9$ $f(-2) = 9$

b. 0 $f(0) = -4(0) + 1 = 1$ $f(0) = 1$

c. 5 $f(5) = -4(5) + 1$ $f(5) = -19$

Vocabulary Bite

The domain* of a relation is the set of *inputs*, also called x-coordinates, of the ordered pairs. The range is a set of *outputs*, also called y-coordinates, of the ordered pairs.

** restricted domain*

$\sqrt{x}$ ✗ neg. numbers are not allowed $\frac{1}{x}$ ✗ cannot divide by 0

Adding and Subtracting Functions

Problem 1

Let $f(x) = 4x + 7$ and $g(x) = \sqrt{x} + x$.

What are $f + g$ and $f - g$?

What are their domains?

$(f + g)(x) =$

$=$

$=$ $(f+g)(x) = 5x + \sqrt{x} + 7$

D: $x \geq 0$

neg. numbers are not allowed

∴ x must be greater than or equal to 0

$(f - g)(x) =$

$=$

$=$ $4x + 7 - \sqrt{x} - x$

$= 3x - \sqrt{x} + 7$ $(f-g)(x) = 3x - \sqrt{x} + 7$

D: $x \geq 0$

Problem 1

You Try:

Let $f(x) = 2x^2 + 8$ and $g(x) = x - 3$.

What are $f + g$ and $f - g$?

What are their domains?

Circle $f + g$. $(2x^2+8)+(x-3)$

$2x^2 + x + 8$ | $2x^2 + x + 5$ | $x^2 + 5$ | $x^2 + 8$

$(f + g)(x) = 2x^2 + x + 5$

Circle $f - g$. $(2x^2+8)-(x-3) = 2x^2+8-x+3$

$2x^2 - x + 11$ | $2x^2 - x + 5$ | $x^2 + 5$ | $x^2 + 11$

The domain of both $f + g$ and $f - g$ is D: all real numbers ✓

$\mathbb{R}$

Multiplying and Dividing Functions

Problem 2

Let $f(x) = x^2 - 9$ and $g(x) = x + 3$.

What are $f \cdot g$ and $\frac{f}{g}$ and their domains?

★ You cannot cancel *terms*.
You can only cancel *factors*.

$\frac{9 \cdot x}{x}$ good $\quad \frac{9 + x}{x}$ bad

$(f \cdot g)(x)$ FOIL

$= x^3 + 3x^2 - 9x - 27$

D: $\mathbb{R}$ all real numbers

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

factor x^2+0x-9

mult -9 | add 0

$-3 \quad 3$

$\frac{f}{g}$

D: $x+3 \neq 0$

$-3 \quad -3$

$x \neq -3$ ✓

A2.L7-6.NOTES

Problem 2

You Try:

Let $f(x) = x - 2$ and $g(x) = x^2 - 7x + 10$ What are $f \cdot g$ and $\frac{f}{g}$ and their domains?

Circle $f \cdot g$. $= f(x) \cdot g(x)$

$(x-2)\cdot(x^2-7x+10)$

$x^2 - 7x + 10 + x - 2$ $\quad (x^2 - 7x + 10)(x - 2)$ $\quad \frac{x^2 - 7x + 10}{x - 2}$

Simplify the product.

$x^3 - 7x^2 + 10x - 2x^2 + 14x - 20 = x^3 - 9x^2 + 24x - 20$

D: $\mathbb{R}$ all real #'s

Circle $\frac{f}{g}$.

$x^2 - 7x + 10 + x - 2$ $\quad (x^2 - 7x + 10)(x - 2)$ $\quad \frac{g}{f}\ \frac{x^2 - 7x + 10}{x - 2}$

Simplify the quotient.

$\frac{1(x-2)}{(x-2)(x-5)} = \frac{1}{x-5}$

D: $x - 5 \neq 0$

D: $x \neq 5$

$\frac{x-2}{x^2-7x+10}$

mult	sum
10	-7
-5 -2	

Write the domain of $f \cdot g$.

D: all real numbers

Write the domain of $\frac{f}{g}$.

D: $x \neq 5$

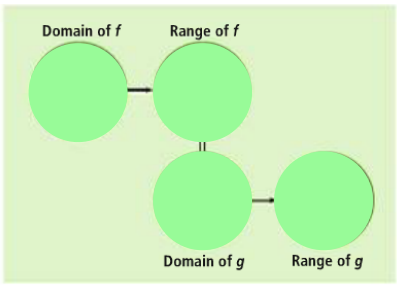
The diagram shows what happens when you apply one function $g(x)$ after another function $f(x)$.

The output from the first function becomes the input for the second function. When you combine two functions as in the diagram, you form a .

composite (adjective) **kum PAHZ it**

Related Words: composite number

Main Idea: Something that is **composite** is made up of more than one thing.



Math Usage: A **composite** function uses the output of one function as the input of a second function.

take note

Key Concept Composition of Functions

The composition of function g with function f is written $g \circ f$ and is defined as $(g \circ f)(x) = g(f(x))$. The domain of $g \circ f$ consists of the x-values in the domain of f for which $f(x)$ is in the domain of g.

$(g \circ f)(x) = g(f(x))$

1. Evaluate $f(x)$ first.
2. Then use $f(x)$ as the input for g.

Function composition is not commutative, since $f(g(x))$ does not always equal $g(f(x))$.

Complete each sentence with the correct form of the word *composite*.

composite composition compose

VERB The musician worked to ? a new piece of music. Compose

ADJECTIVE A ? number has more than two factors. Composite

NOUN The poster was a ? of photos and famous quotes. Composition

Composing Functions

Problem 3

Let $f(x) = x - 5$ and $g(x) = x^2$. What is $(g \circ f)(-3)$?

Method 1

$(g \circ f)(-3)$ — composed of

$g(f(-3))$

Step 1 find value $f(-3)$

$f(x) = x - 5$

$f(-3) = -3 - 5$

$f(-3) = -8$

$g(f(-3)) = g(-8)$

Step 2 find value $g(-8)$ $g(x) = x^2$

$g(-8) = (-8)^2$

$g(-8) = 64$

$(g \circ f)(-3) = 64$

Composing Functions

Problem 3

You Try:

Let $f(x) = x - 5$ and $g(x) = x^2$. What is $(f \circ g)(-3)$? $= f(g(-3))$ Step 1

Find $g(-3)$.

Use $g(-3)$ as the input for $f(x)$.

$(f \circ g)(-3) = f(g(-3)) = f(\quad) = \quad = \quad$

Using Composite Functions

Problem 4

You have a coupon good for $5 off the price of any large pizza. You also get a 10% discount on any pizza if you show your student ID. How much more would you pay for a large pizza if the cashier applies the coupon first?

Know	Need	Plan
The coupon value and the discount rate	The difference between the results of applying the discount or coupon first	• Compose two functions in two ways. • Then find the difference in their results.

coupon discount

Step 1 Find functions C and D that model the cost of a large pizza.

Let x = the price of a large pizza.

Cost using the coupon: $C(x) = x - 5$

10% 0.10 90% 0.90

Cost using the 10% discount: $D(x) = x - 0.1x = 0.9x$

Step 2 Compose the functions to apply the discount and then the coupon.

$(C \circ D)(x) = C(D(x))$ Apply the discount, $D(x)$, first.

$= C(0.9x) = 0.9x - 5$

$C(x) = x - 5$

Step 3 Compose the functions to apply the coupon and then the discount.

$(D \circ C)(x) = D(C(x))$ Apply the coupon, $C(x)$, first.

$= D(x - 5) = 0.9(x - 5) = 0.9x - 4.5$

Step 4 Subtract the functions to find how much more you would pay if the cashier applies the coupon first.

$(D \circ C)(x) - (C \circ D)(x) = (0.9x - 4.5) - (0.9x - 5)$

$= -4.5 + 5$

$= 0.5$

Using Composite Functions

Problem 4

You Try:

A store is offering a 15% discount on all items. Also, employees get a 20% employee discount. Write composite functions.
Model taking the 15% discount and then the 20% discount.
Model taking the 20% discount and then the 15% discount.

Let x be the price of an item. Write functions to model each discount.

$C(x)$ = cost using the storewide discount $= x -$ ____ $x =$ ____ x

$D(x)$ = cost using the employee discount $= x -$ ____ $x =$ ____ x

Draw a line from each composition to its rule.

$(D \circ C)(x)$	$0.85(0.8x)$
$(C \circ D)(x)$	$0.8(0.85x)$

Simplify each function.

$(D \circ C)(x) =$ ____ $(C \circ D)(x) =$ ____

If you were an employee, which discount would you take first? Why?

_____

_____

L7-6 HOMEWORK

Function Operations

Complete #9-35 [multiples of 3]

Let $f(x) = 7x + 5$ and $g(x) = x^2$. Perform each function operation and then find the domain of the result.

9. $(f + g)(x)$ **10.** $(f - g)(x)$ **11.** $(g - f)(x)$

12. $(f \cdot g)(x)$ **13.** $\frac{f}{g}(x)$ **14.** $\frac{g}{f}(x)$

Let $f(x) = 2 - x$ and $g(x) = \frac{1}{x}$. Perform each function operation and then find the domain of the result.

15. $(f + g)(x)$ **16.** $(f - g)(x)$ **17.** $(g - f)(x)$

18. $(f \cdot g)(x)$ **19.** $\frac{f}{g}(x)$ **20.** $\frac{g}{f}(x)$

Let $f(x) = 2x^2 + x - 3$ and $g(x) = x - 1$. Perform each function operation and then find the domain.

21. $(f + g)(x)$ **22.** $(f - g)(x)$ **23.** $(g - f)(x)$

24. $(f \cdot g)(x)$ **25.** $\frac{f}{g}(x)$ **26.** $\frac{g}{f}(x)$

Let $g(x) = 2x$ and $h(x) = x^2 + 4$. Find each value or expression.

27. $(h \circ g)(1)$ **28.** $(h \circ g)(-5)$ **29.** $(h \circ g)(-2)$

30. $(g \circ h)(-2)$ **31.** $(g \circ h)(0)$ **32.** $(g \circ h)(a)$

33. $(g \circ g)(a)$ **34.** $(h \circ h)(a)$ **35.** $(h \circ g)(a)$